Base Vectors

Q: *You said earlier that* ***vector*** *quantities (either discrete or field) have* ***both*** *and magnitude and* ***direction****. But how do we* ***specify*** *direction in 3-D space? Do we use* ***coordinate*** *values (e.g., x, y, z)??*

A: It is very important that you understand that **coordinates only** allow us to specify **position** in 3-D space. They **cannot** be used to specify **direction**!

The most convenient way for us to specify the direction of a vector quantity is by using a well-defined **orthornormal set** of vectors known as **base vectors**.

Recall that an orthonormal set of vectors, say $\hat{a}_1, \hat{a}_2, \hat{a}_3$, have the following properties:

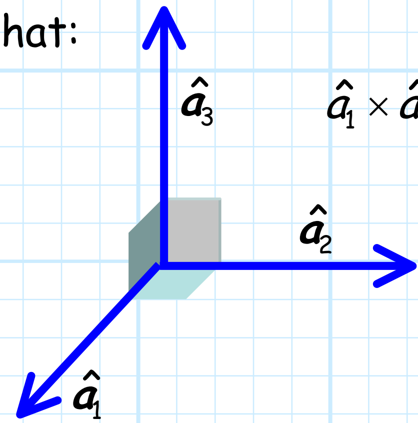
1. Each vector is a **unit** vector:

$$\hat{a}_1 \cdot \hat{a}_1 = \hat{a}_2 \cdot \hat{a}_2 = \hat{a}_3 \cdot \hat{a}_3 = 1$$

2. Each vector is mutually **orthogonal**:

$$\hat{a}_1 \cdot \hat{a}_2 = \hat{a}_2 \cdot \hat{a}_3 = \hat{a}_3 \cdot \hat{a}_1 = 0$$

Additionally, a set of base vectors $\hat{a}_1, \hat{a}_2, \hat{a}_3$ must be arranged such that:

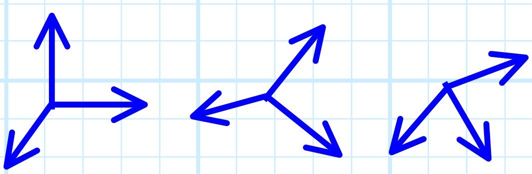


$$\hat{a}_1 \times \hat{a}_2 = \hat{a}_3 , \quad \hat{a}_2 \times \hat{a}_3 = \hat{a}_1 , \quad \hat{a}_3 \times \hat{a}_1 = \hat{a}_2$$

An orthonormal set with this property is known as a **right-handed** system.

> **All** base vectors $\hat{a}_1, \hat{a}_2, \hat{a}_3$ must form a **right-handed, orthonormal** set.

Recall that we use **unit vectors** to define **direction**. Thus, a set of base vectors defines three distinct directions in our 3-D space!



Q: *But, **what three** directions do we use?? I remember that you said there are an **infinite** number of possible **orientations** of an orthonormal set!!*

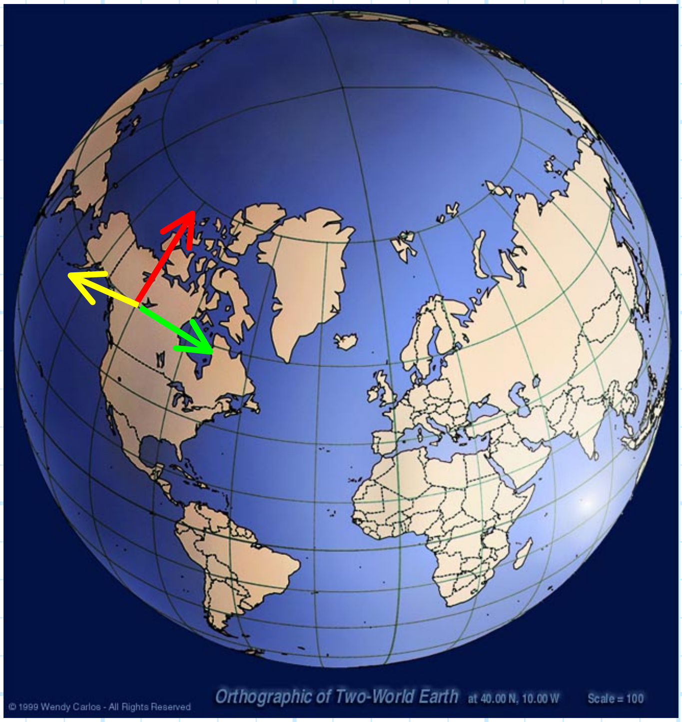
A: We will define several systematic, mathematically **precise methods** for defining the orientation of base vectors. Generally speaking, we will find that the orientation of these base vectors will **not be fixed**, but will in fact vary with **position** in space (i.e., as a function of coordinate values)!

Essentially, we will define at **each** and every point in space a **different** set of basis vectors, which can be used to uniquely define the direction of any vector quantity **at that point**!

> **Q:** *Good golly! Defining a **different** set of base vectors for **every** point in space just seems **dad-gum confusing.** Why can't we just **fix** a set of base vectors such that their orientation is the **same** at **all** points in space?*

A: We will in fact study **one** method for defining base vectors that **does** in fact result in an othonormal set whose orientation is **fixed**—the same at **all** points in space (Cartesian base vectors).

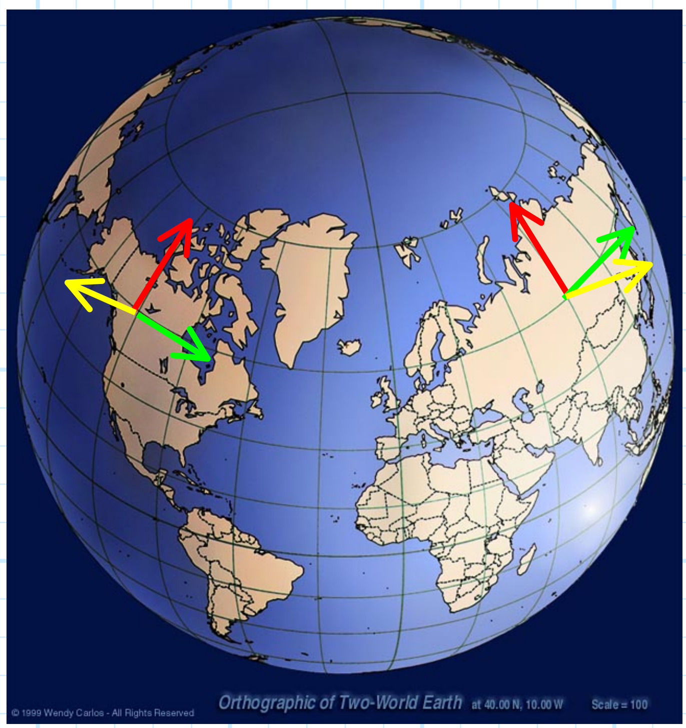
However, we will study **two other** methods where the orientation of base vectors is **different** at all points in space (spherical and cylindrical base vectors). We use these two methods to define base vectors because for **many** physical problems, it is actually **easier** and **wiser** to do so!



For example, consider how we define direction on **Earth**: North/South, East/West, Up/Down.

Each of these directions can be represented by a **unit vector**, and the three unit vectors together form a set of **base vectors**.

Think about, however, how these base vectors are oriented! Since we live on the surface of a **sphere** (i.e., the Earth), it makes sense for us to orient the base vectors with **respect to the spherical surface**.



What this means, of course, is that **each location** on the Earth will orient its "base vectors" differently. This orientation is thus **different** for every point on Earth—a method that makes **perfect sense**!